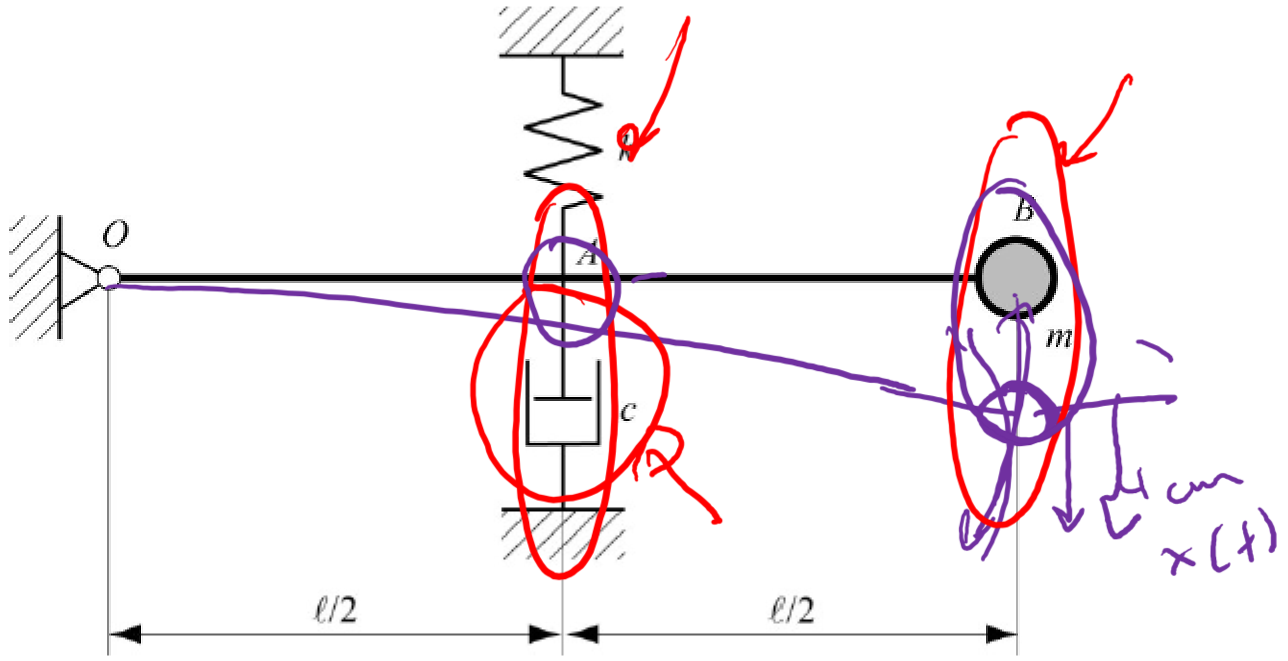
EPFL

Semaine 3
Doutes

ME-332 – Mécanique Vibratoire

École
polytechnique
fédérale
de Lausanne

Exo 2.2



$$x(t) = X \cdot e^{-\lambda t} \cdot \cos(\omega_1 t - \varphi)$$

$$v(t) = \frac{\partial x}{\partial t} = -X\, e^{-\lambda t} \left[\lambda \cos(\omega_1 t - \varphi) + \omega_1 \cdot \sin(\omega_1 t - \varphi)\right]$$

$t=0$:

$$v(t=0) = 0 = -X\left[\lambda \cdot \cos\varphi - \omega_1 \sin\varphi\right] \Rightarrow \tan\varphi = \frac{\lambda}{\omega_1}$$

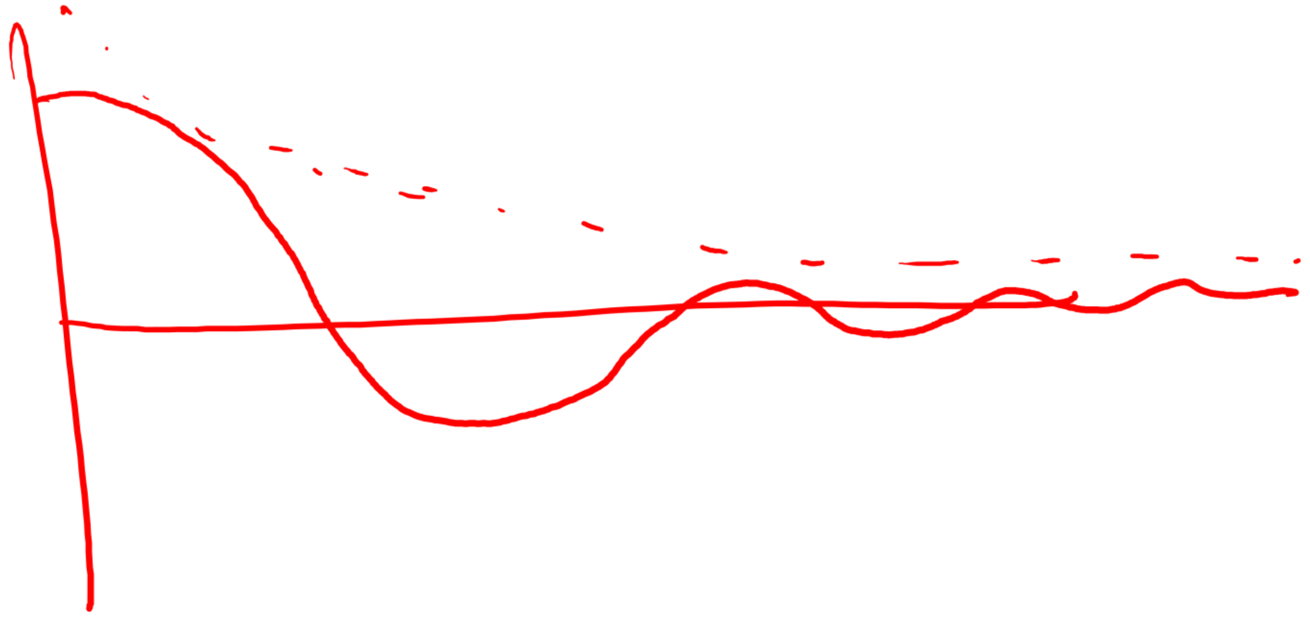
$$\eta \ll 1 \,;\ \eta^2 \lll 1 \Rightarrow \omega_1 \sim \omega_0 \Rightarrow \tan\varphi = \frac{\lambda}{\omega_1} \sim \frac{\lambda}{\omega_0} = \eta$$

$$\tan\varphi \sim \eta \ll 1 \Rightarrow \varphi \sim 0$$

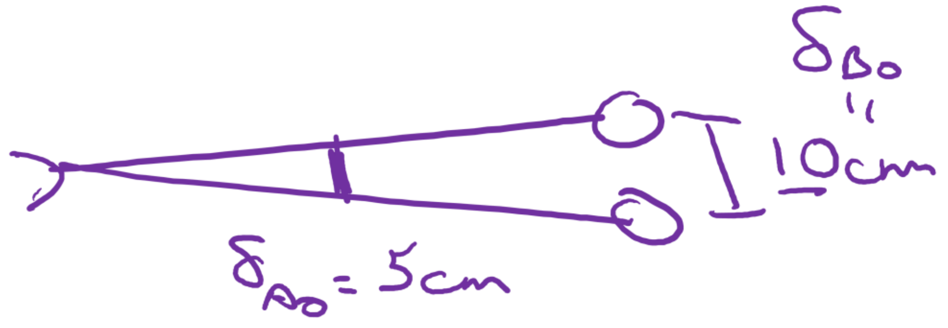
$$x(t=0) = X \cdot \underbrace{\cancel{e^{-\lambda 0}}}_{1} \cdot \underbrace{\cancel{\cos(\omega_1 0 - 0)}}_{1} = X = \delta_A$$

$$P_{dissip\acute{e}e} = F_{am} \cdot V_{am} = c \cdot V_{am}^2 = \frac{c}{4} V_m^2$$

$$P_{dis} = \frac{1}{4} c \cdot X^2 e^{-2\lambda t} \left[\lambda \cos(\omega_1 t) + \omega_1 \sin(\omega_1 t)\right]^2$$



Équil. Statique



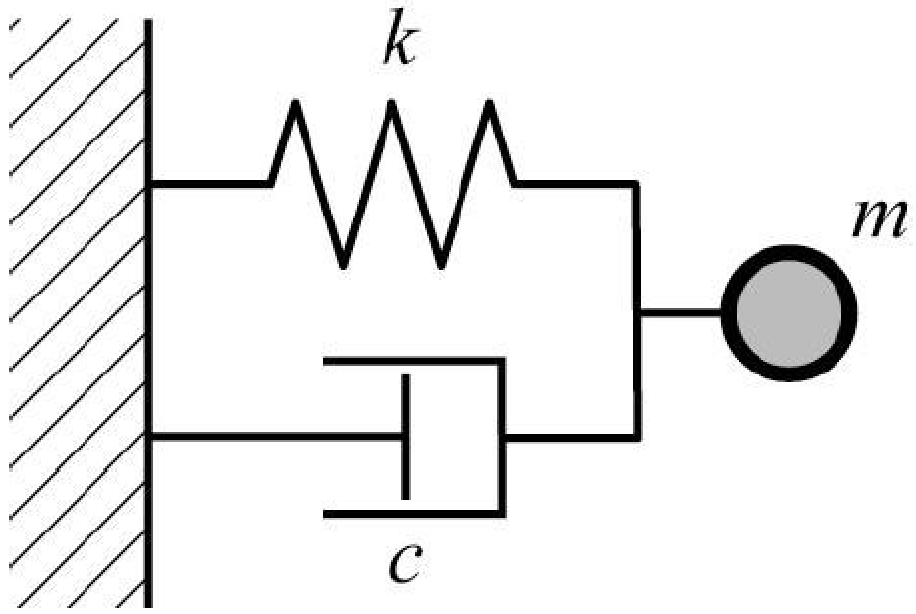
Exo 2.1

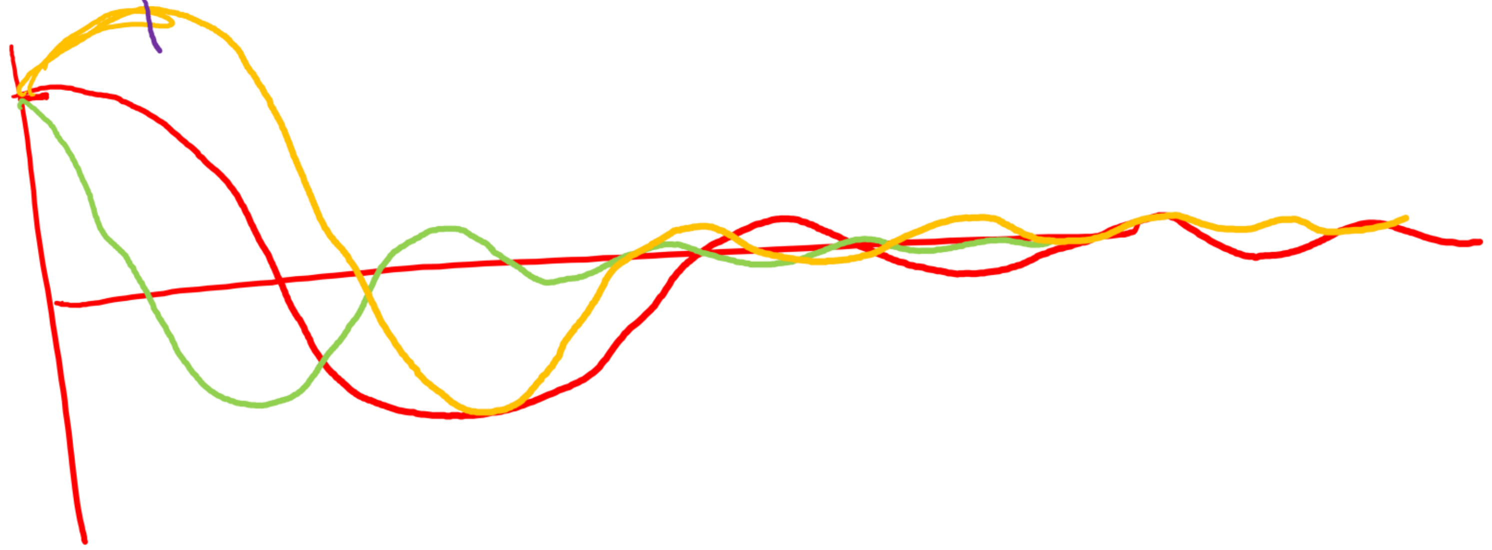
L'amplitude de la cinquième oscillation de l'amortisseur dans la Figure 2.1.1 est égale à 25% de celle de la première.

Calculer l'amortissement relatif η et absolu c pour une rigidité k de 3500 N/m et une masse m de 5 kg,

a) avec l'approximation que la pulsation propre ω_1 est égale à la pulsation ω_0 du système conservatif associé,

b) sans cette approximation.





$$x(t) = X \cdot e^{-\lambda t} \cdot \cos(\omega_1 t - \varphi)$$

$$\cos(\omega_1 t_0 - \varphi) = 1$$

$$\omega_1 t_0 - \varphi = 0 \Rightarrow t_0 = \frac{\varphi}{\omega_1}$$

$$T_1 = \frac{2\pi}{\omega_1}$$

1er max → t_0

2em max → $t_0 + T_1$

3em max → $t_0 + 2T_1$

$$F = F_0 \cdot \cos(\omega t)$$

$$\tilde{F} = F_0 \cdot e^{j\omega t} \rightarrow \tilde{x} = X_0 \cdot e^{j(\omega t - \varphi)}$$

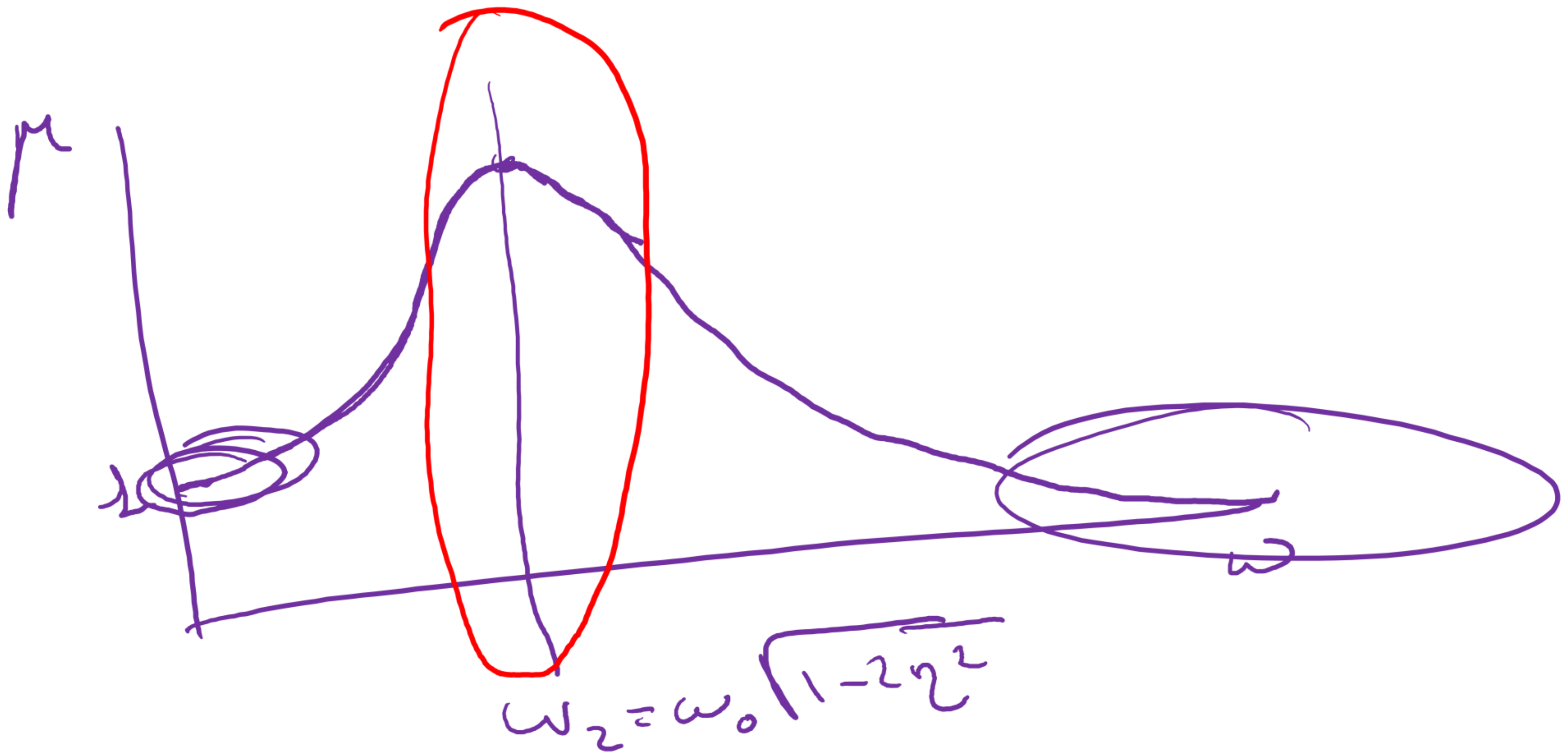
$$\dot{\tilde{x}} = j\omega X_0 e^{j(\omega t - \varphi)}$$

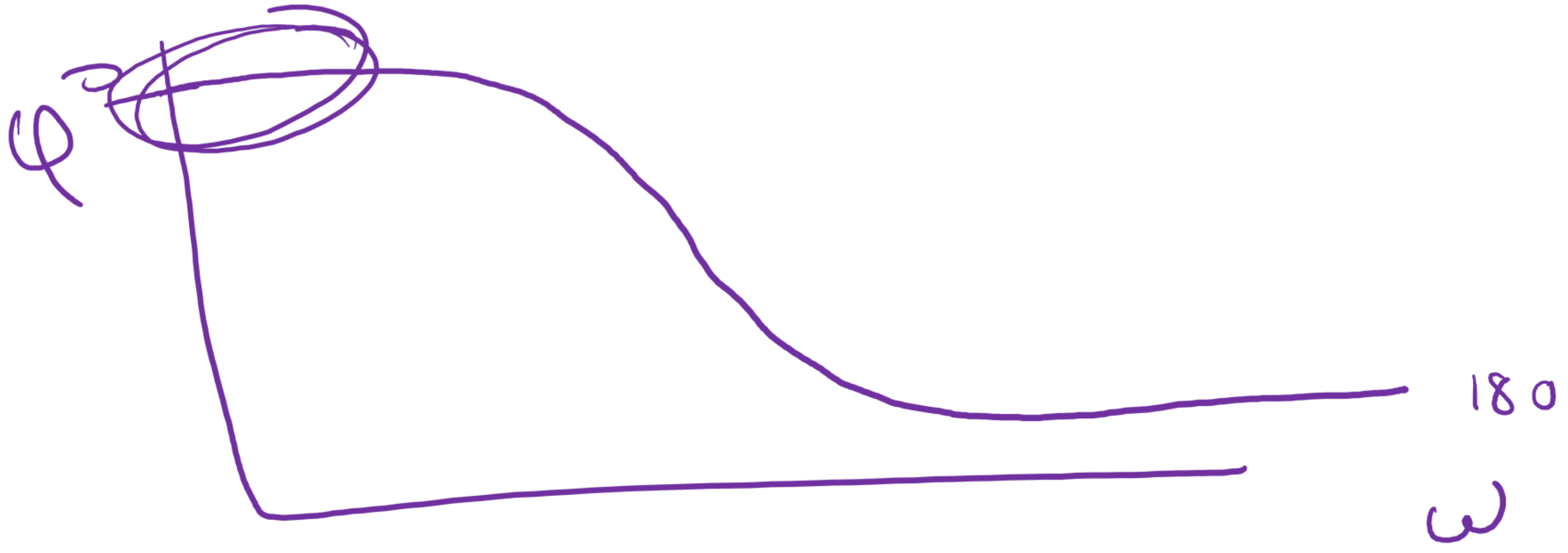
$$\ddot{\tilde{x}} = -\omega^2 X_0 e^{j(\omega t - \varphi)}$$

$$m\ddot{x} + c\dot{x} + kx = F$$

$$-\omega^2 m X_0 e^{\cancel{j(\omega t}-\varphi)} + j\omega c X_0 e^{\cancel{j(\omega t}-\varphi)} + k X_0 e^{\cancel{j(\omega t}-\varphi)} = F_0 \cancel{e^{j\omega t}}$$

$$X_0 e^{-j\varphi}(k - m\omega^2 + j\omega c) = F_0$$





Semaine 3
QCMs

ME-332 – Mécanique Vibratoire

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Plan du cours

Week	Jours	Partie du cours	Exercices	Homework
1	09.09//11.09	Intro et oscillateur 1D libre conservatif	Série 1	Leçon 1.1&1.2
2	18.09	Oscillateur 1D libre dissipatif	Série 2	Leçon 2.1&2.2
3	23.09//25.09	Régime permanent harmonique (complexe)	Série 3	Leçon 3.1&3.2
4	30.09//02.10	Régime permanent périodique (série de Fourier)	Série 4	Leçon 4.1&4.2
5	07.10//09.10	Régime forcé général (transf. Laplace ou Fourier)	Série 5	Leçon 5.1&5.2
6	14.10//16.10	Systèmes à 2 DDL conservatifs	Série 6	Leçon 6.1&6.2
BREAK!!!				
7	28.10//30.10	Récapitulatif // séance de QCMs	**MIDTERM**	
8	04.11//06.11	Systèmes à 2DDL dissipatifs, osc. de Frahm	Série 7	Leçon 7.1&7.2
9	11.11//13.11	Systèmes à n DDL, conservatif	Série 8a	Leçon 8.1&8.2
10	18.11//20.11	Systèmes à n DDL, conservatif	Série 8b	
11	25.11//27.11	Systèmes à n DDL, dissipatif	Série 9	Leçon 9.1&9.2
12	02.12//04.12	Systèmes à n DDL, forcé	Série 10	Leçon 10.1&10.2
13	09.12//11.12	Systèmes continus: barres & poutres	Série 11	Leçon 11.1&11.2
14	16.12//18.12	Récapitulatif // séance de QCMs		

Before we start… let's connect!

- Smartphones
  - iPhone
  - Android

  Download and install the app "*PointSolutions*"

- Computers (or Windows Phones)
  - Go to www.responseware.eu

- Choose **EUROPE Server** (if asked)
- Join session – **ME332**
- When asked – **do not input your name. Enter as anonymous.**

$x(0) = \delta, \dot{x}(0) = 0$. À quelle fréquence bouge?

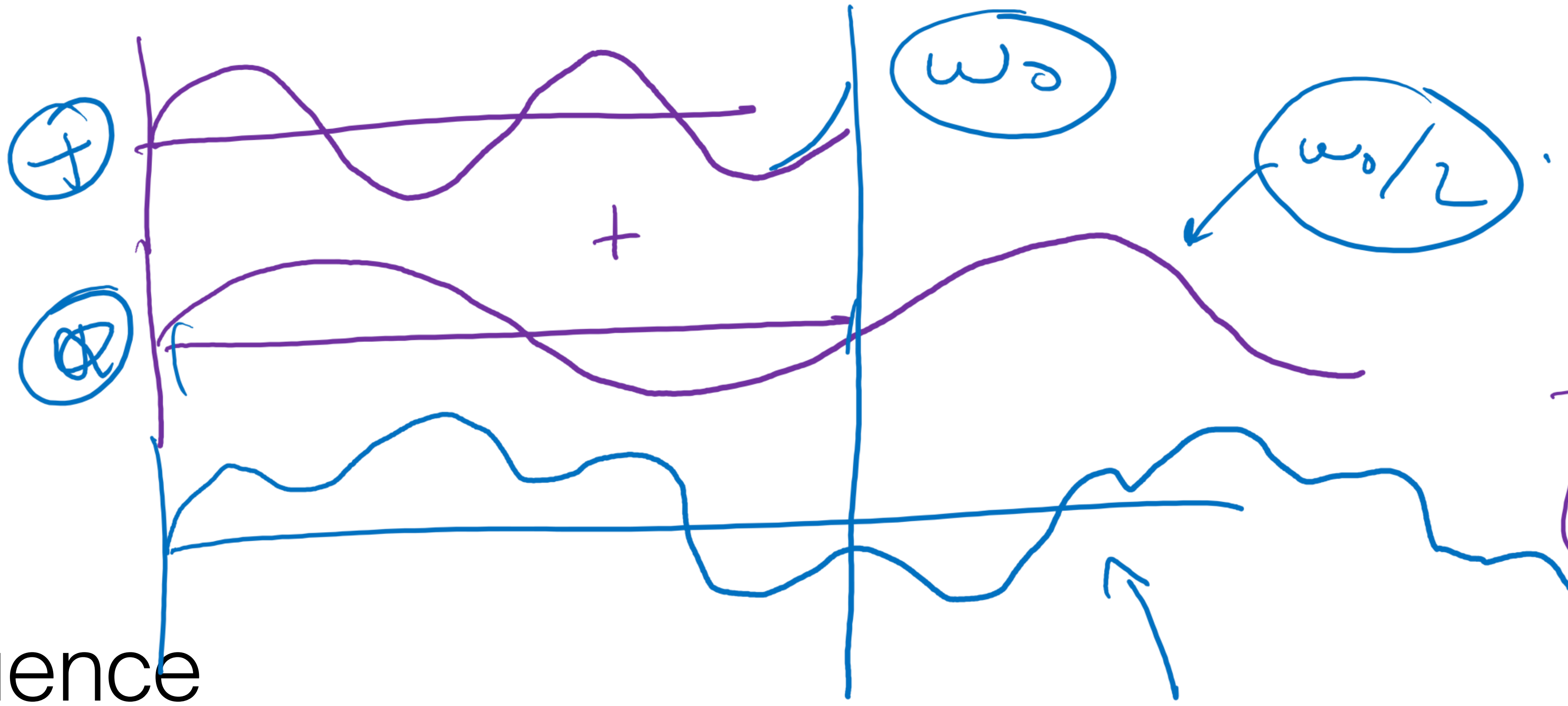
A. ω_0

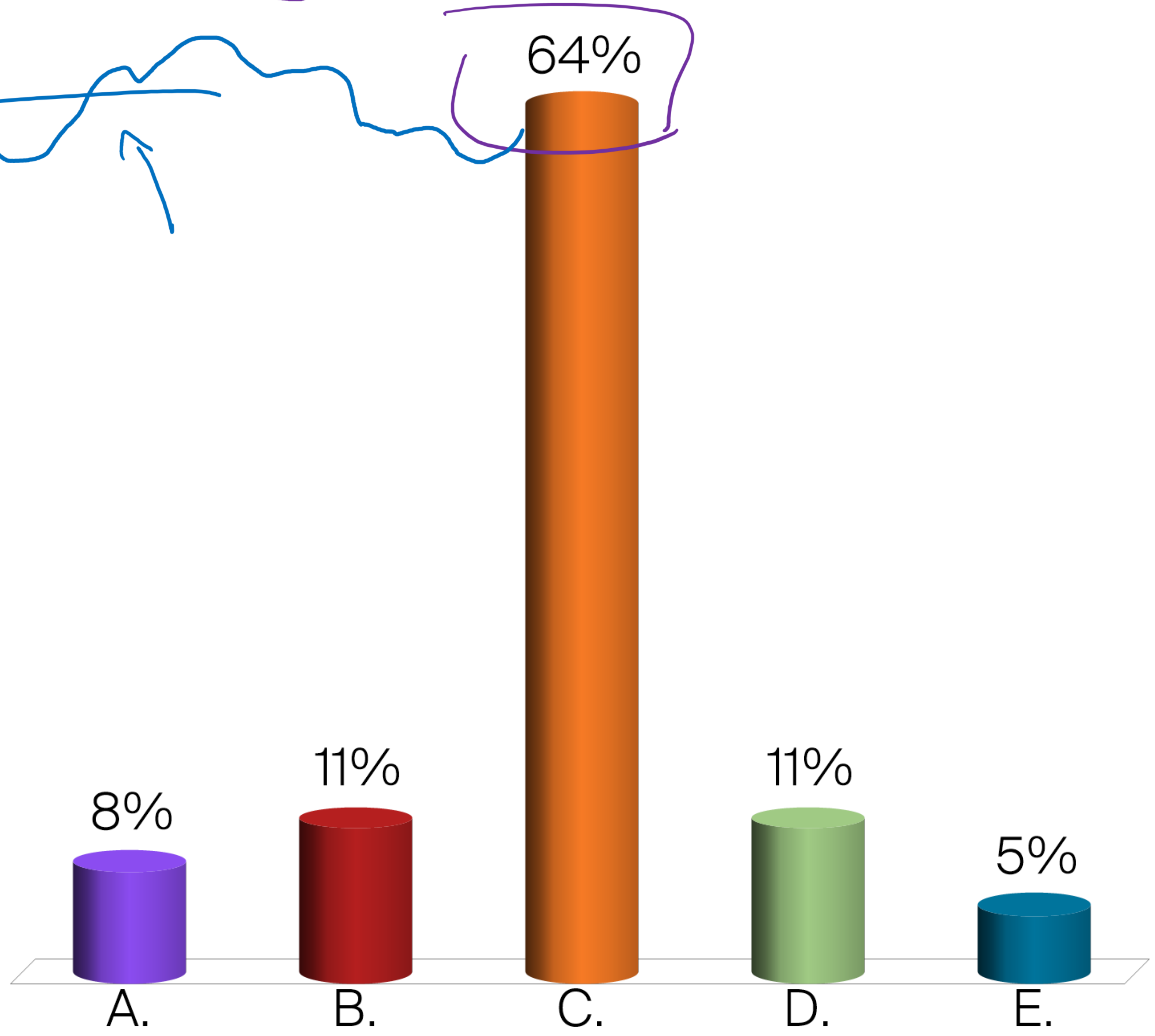
B. ω_1

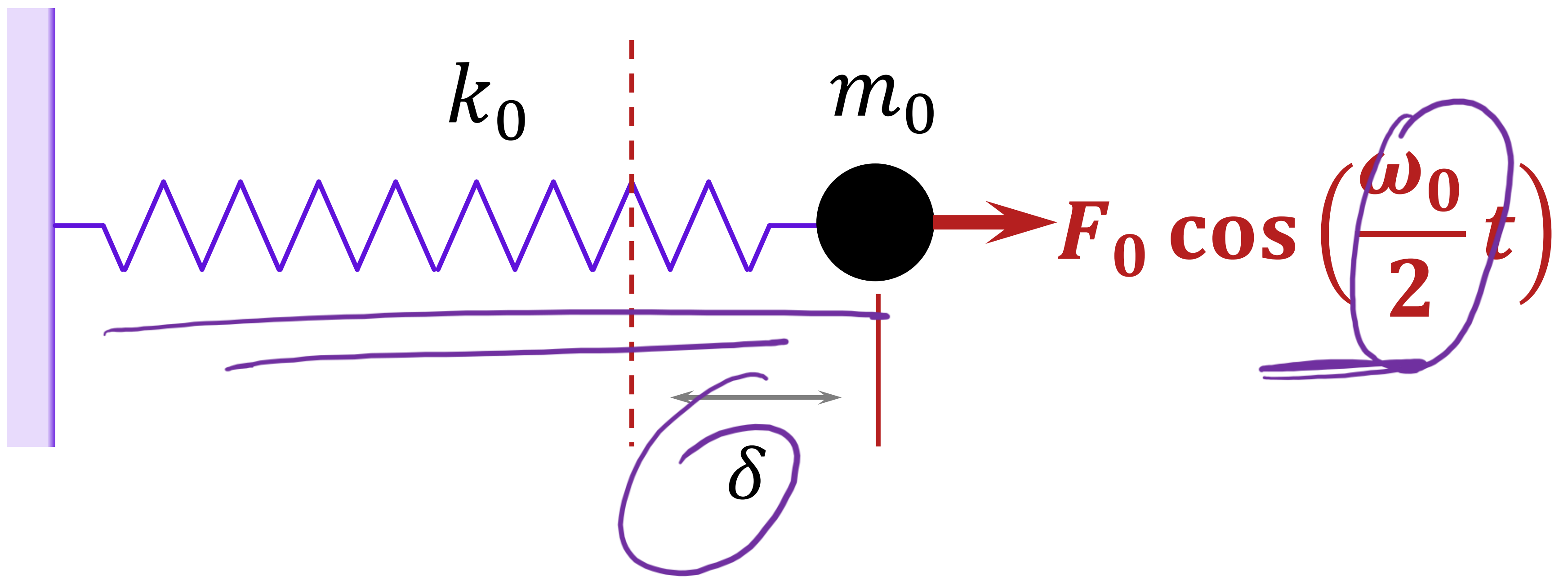
C. $\frac{\omega_0}{2}$

D. Une autre fréquence

E. On ne peut pas savoir







EPFL $x(0) = \delta, \dot{x}(0) = 0$. À quelle fréquence bouge?

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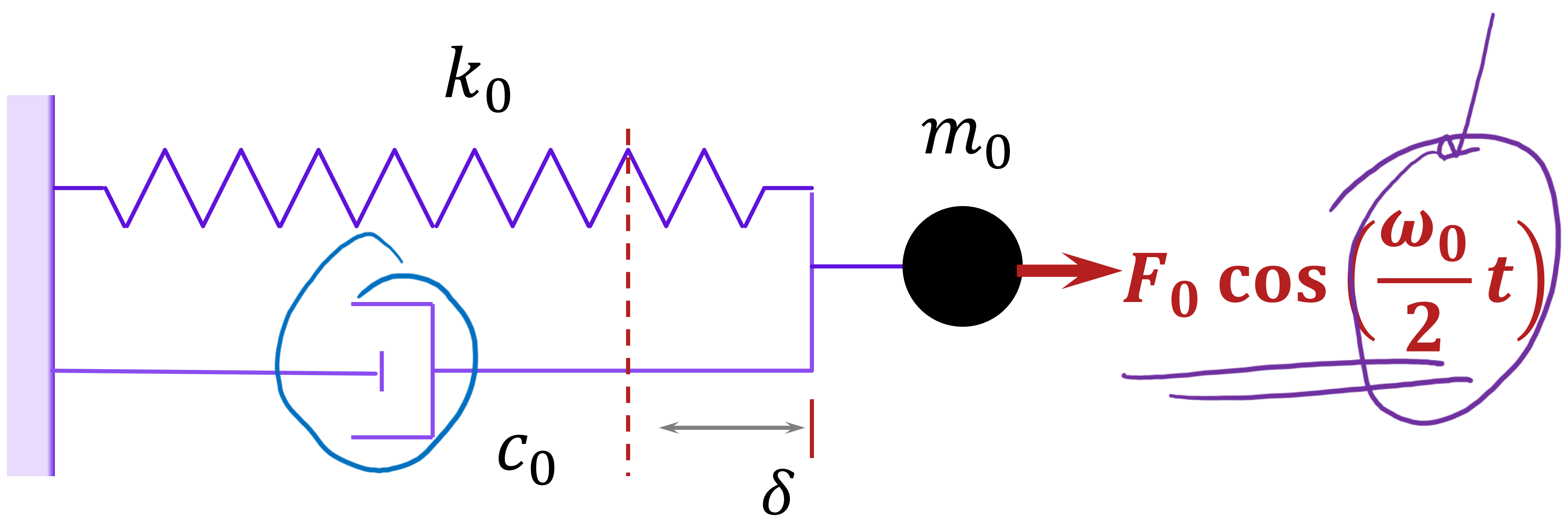
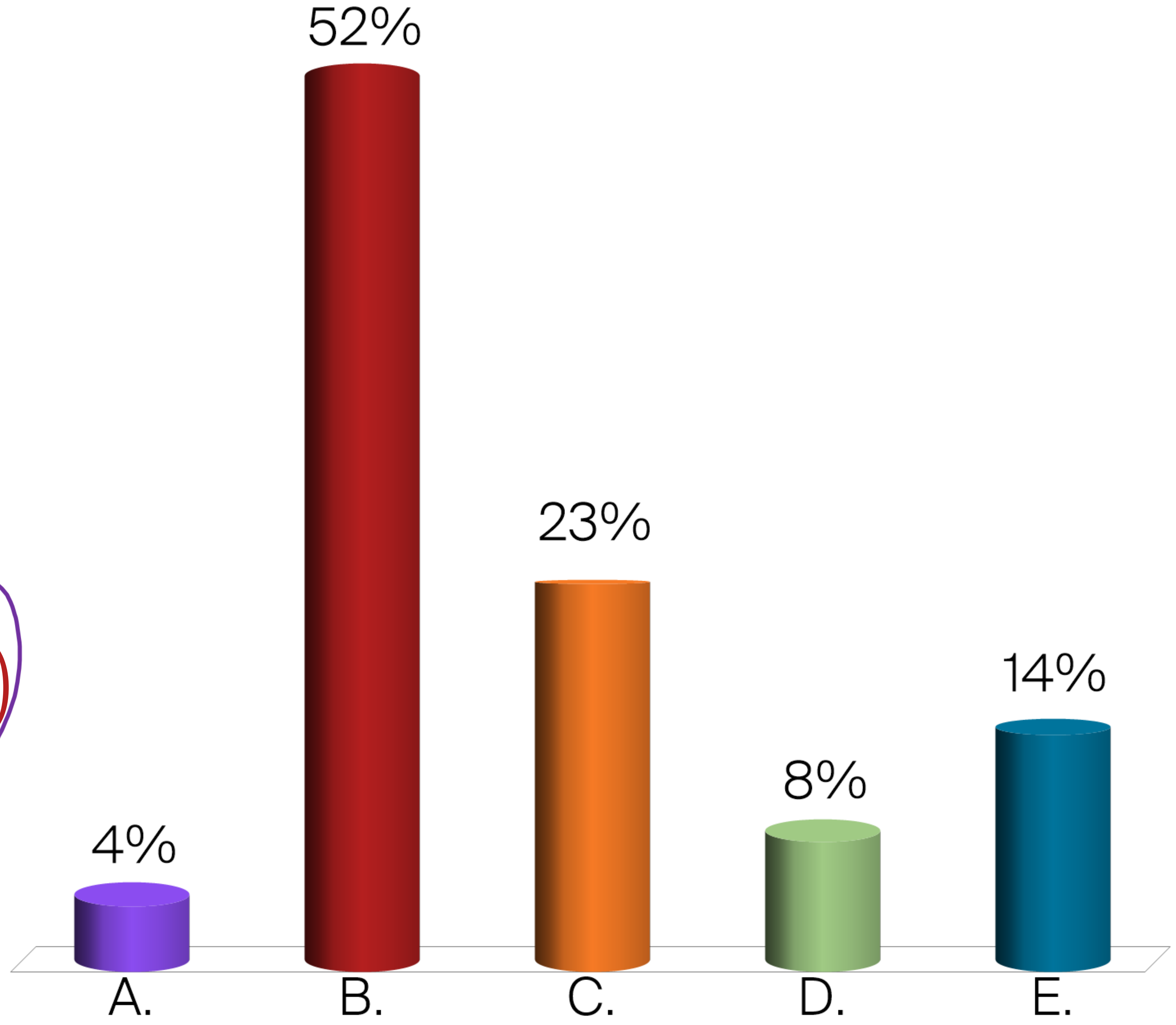


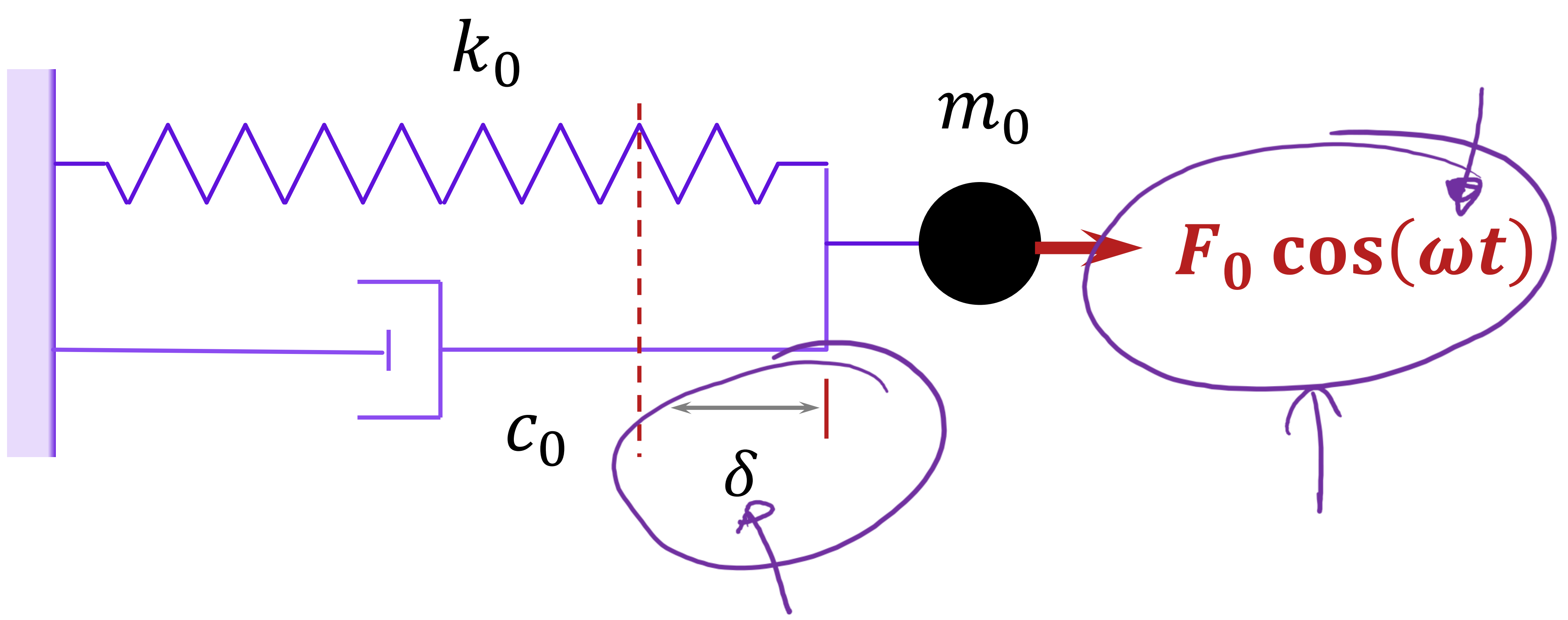


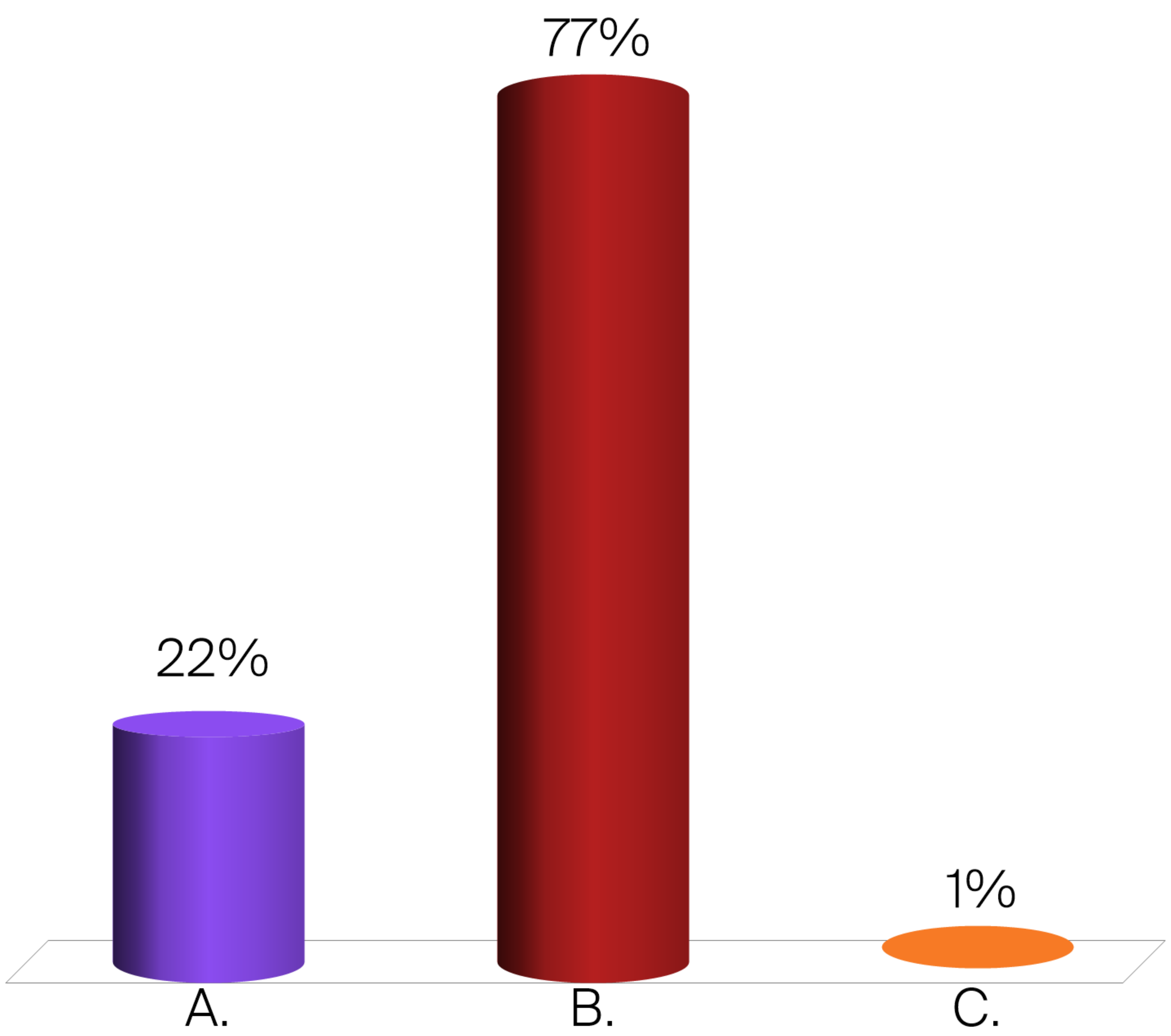
Régime permanent – $x(t)$ dépend des C.I.?

$\hookrightarrow x(0)$
$\dot{x}(0)$

$m\ddot{x} + c\dot{x} + kx = F(t)$

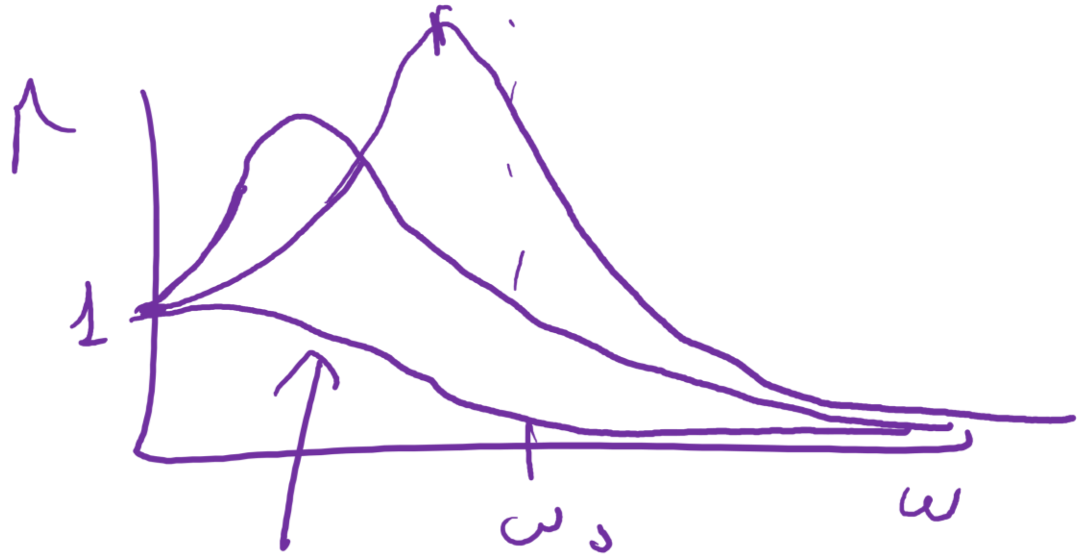
A. Oui
B. Non
C. On ne peut pas savoir





À quelle fréquence on maximise l'amplitude?

η??



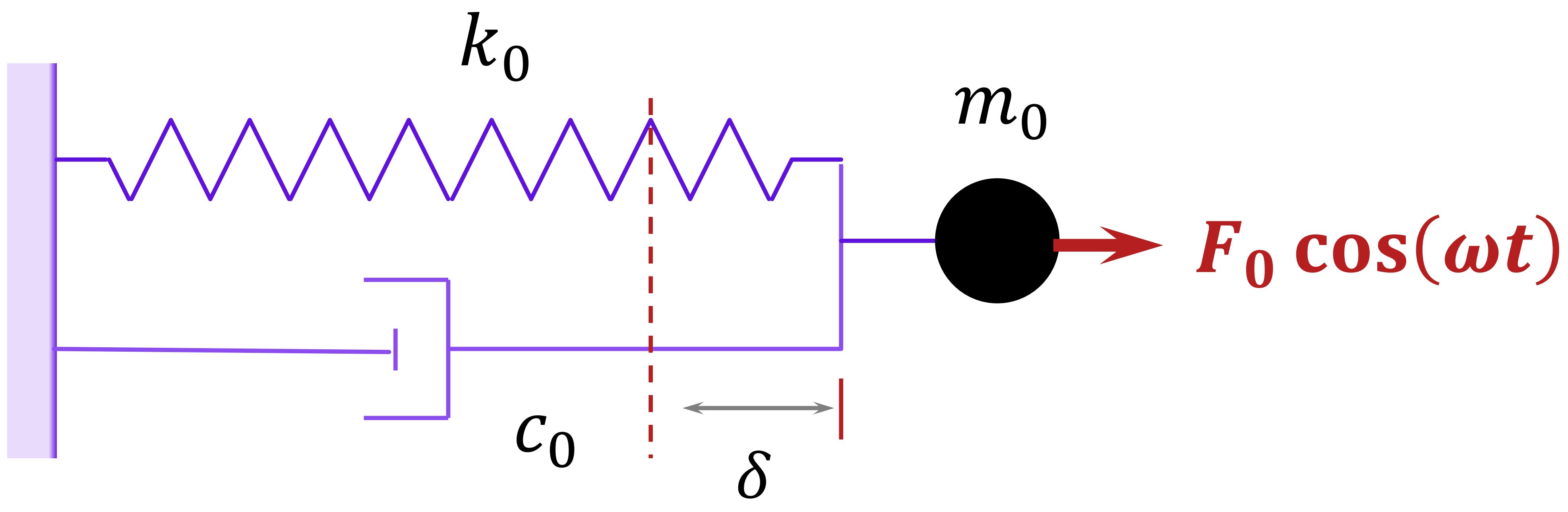
A. ω_0

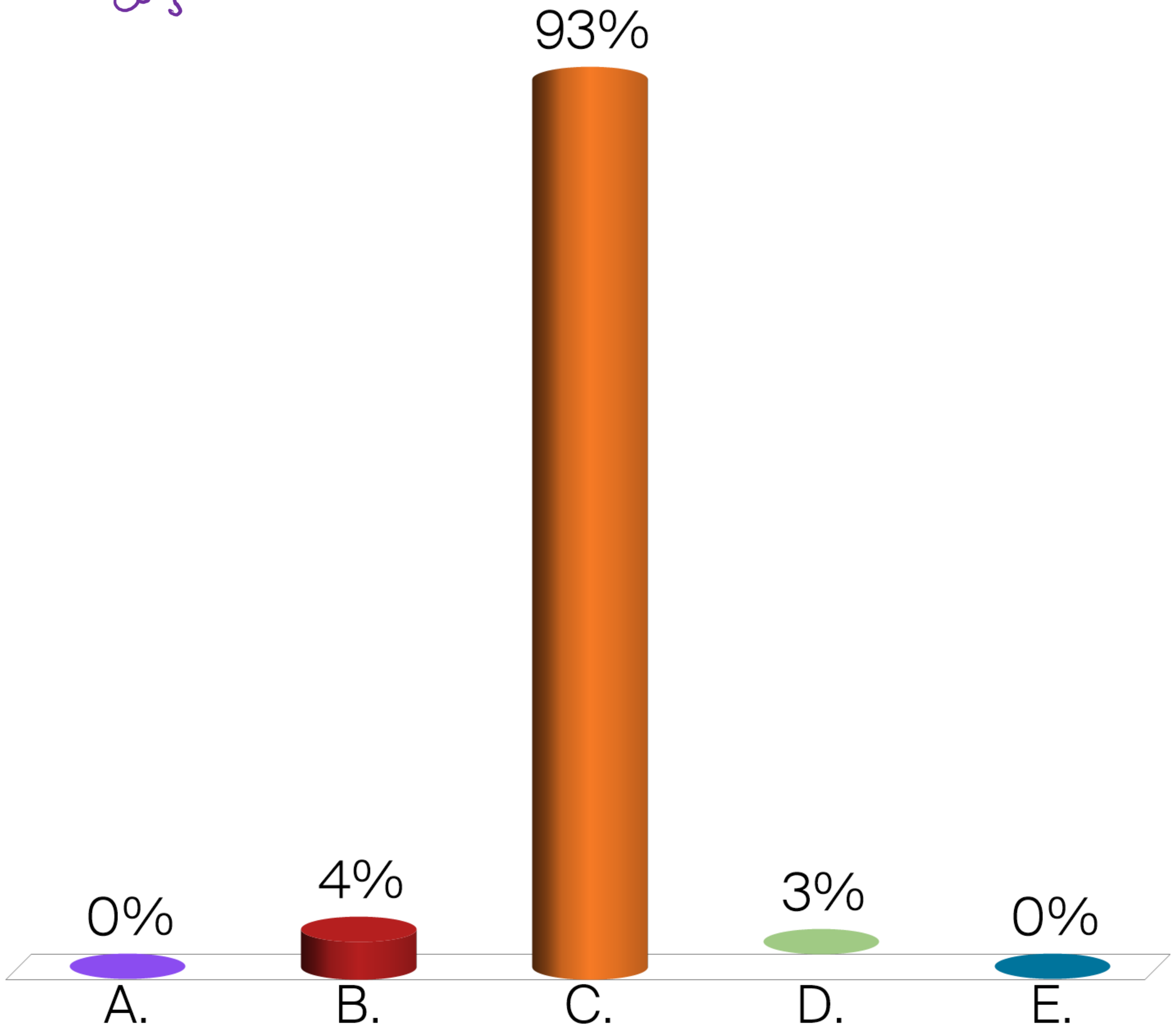
B. $\omega_1 = \omega_0\sqrt{1-\eta^2}$

C. $\omega_2 = \omega_0\sqrt{1-2\eta^2}$ ← $\eta < \frac{1}{\sqrt{2}}$

D. $\omega_3 = \frac{\omega_0}{\sqrt{1-2\eta^2}}$

E. On ne peut pas savoir





À quelle fréquence on maximise la vitesse?

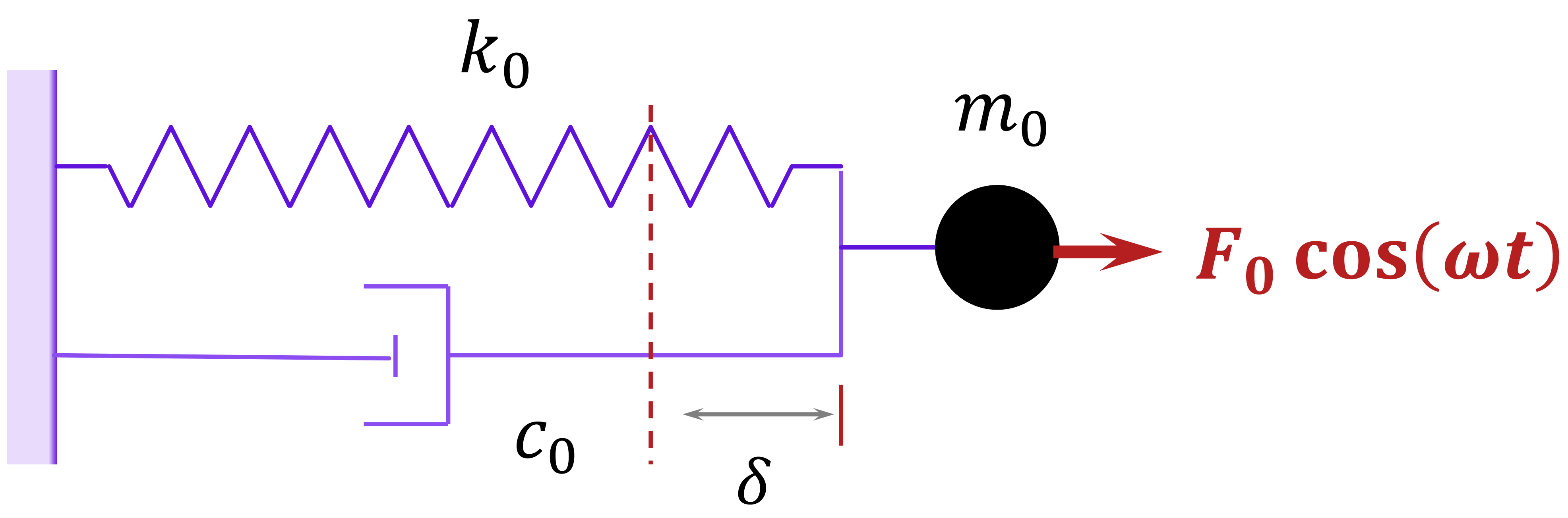
A. ω_0

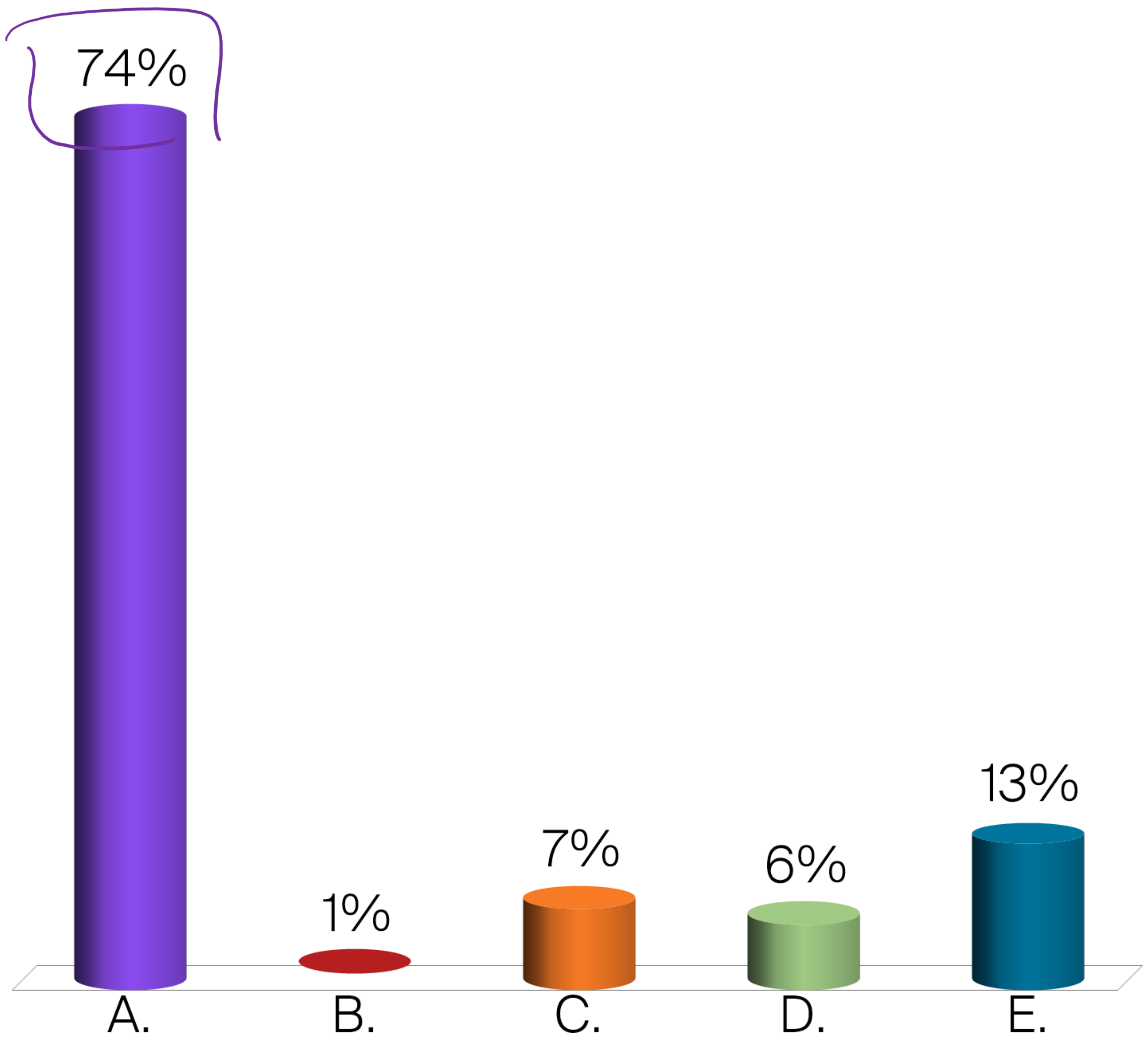
B. $\omega_1 = \omega_0\sqrt{1-\eta^2}$

C. $\omega_2 = \omega_0\sqrt{1-2\eta^2}$

D. $\omega_3 = \frac{\omega_0}{\sqrt{1-2\eta^2}}$

E. On ne peut pas savoir





À quelle fréquence on maximise l'accélération?

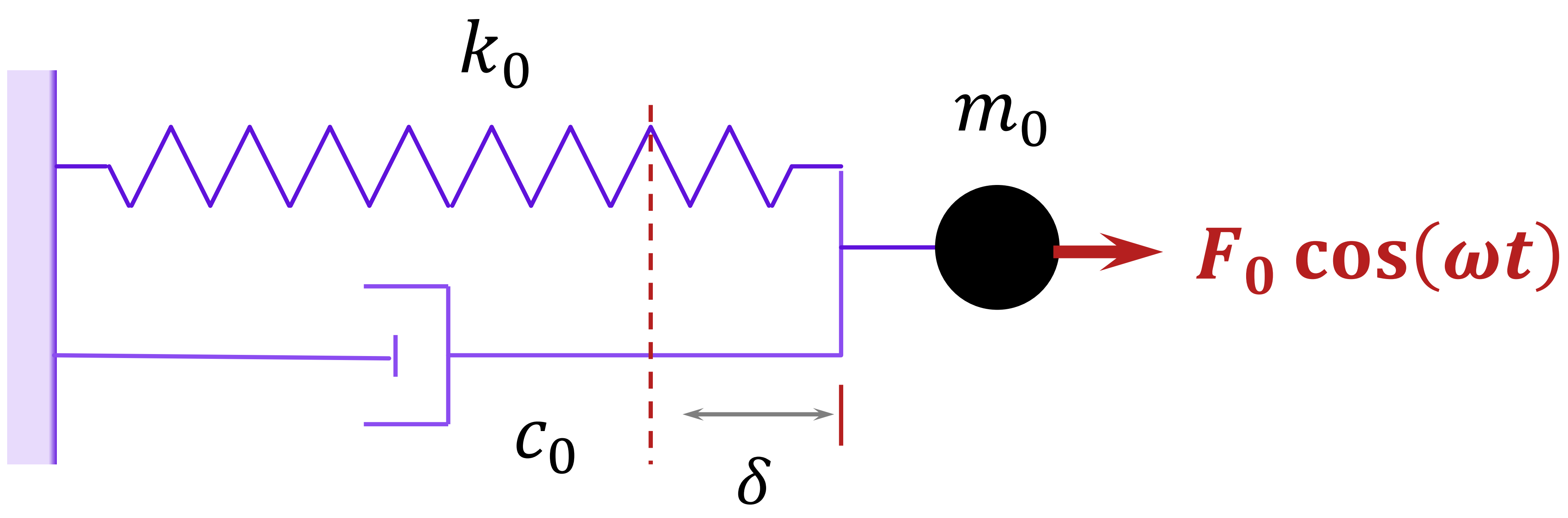
A. ω_0

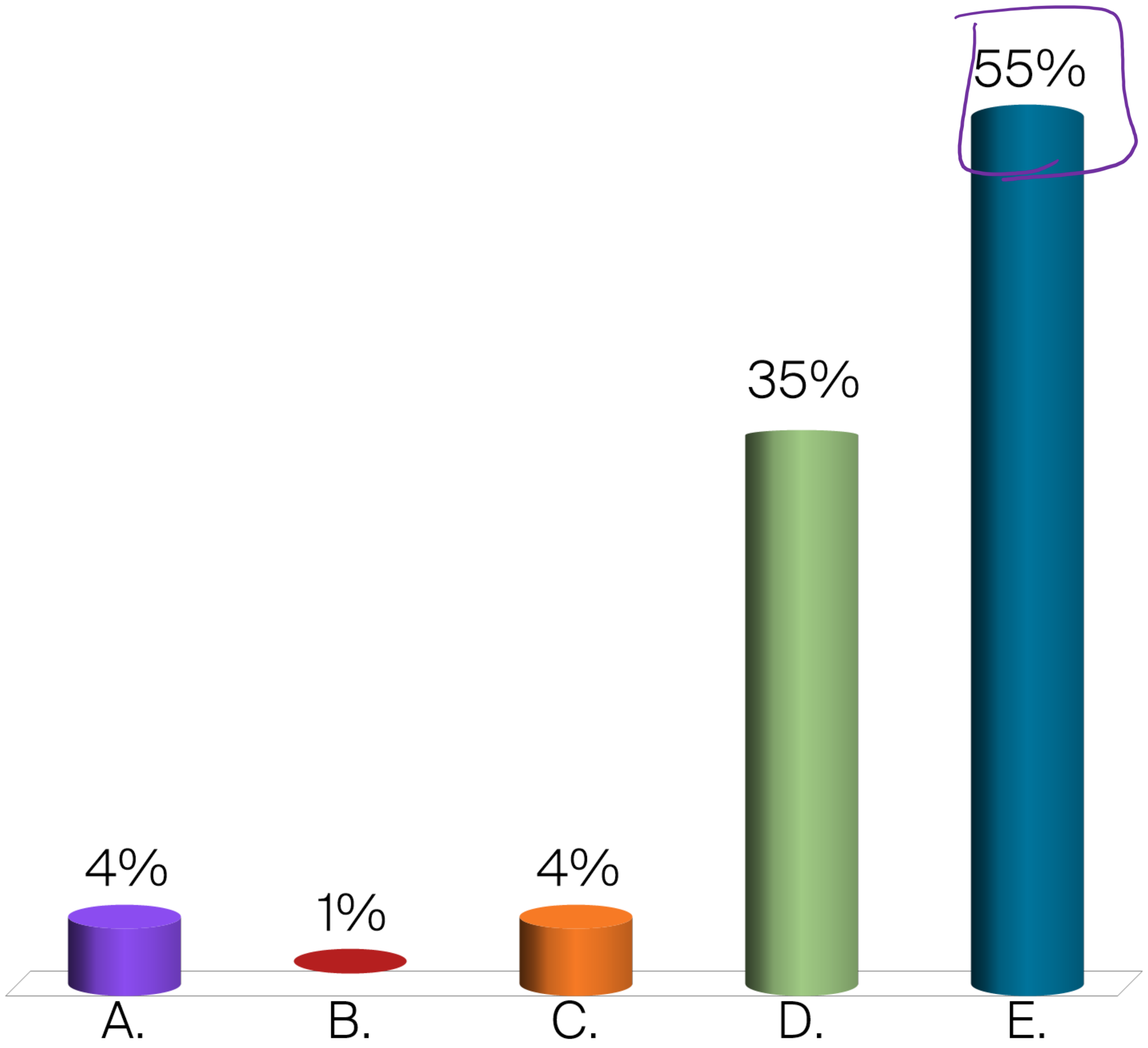
B. $\omega_1 = \omega_0\sqrt{1-\eta^2}$

C. $\omega_2 = \omega_0\sqrt{1-2\eta^2}$

D. $\omega_3 = \frac{\omega_0}{\sqrt{1-2\eta^2}}$

E. On ne peut pas savoir





Régime permanent – $x(t)$ si $\omega \to 0$

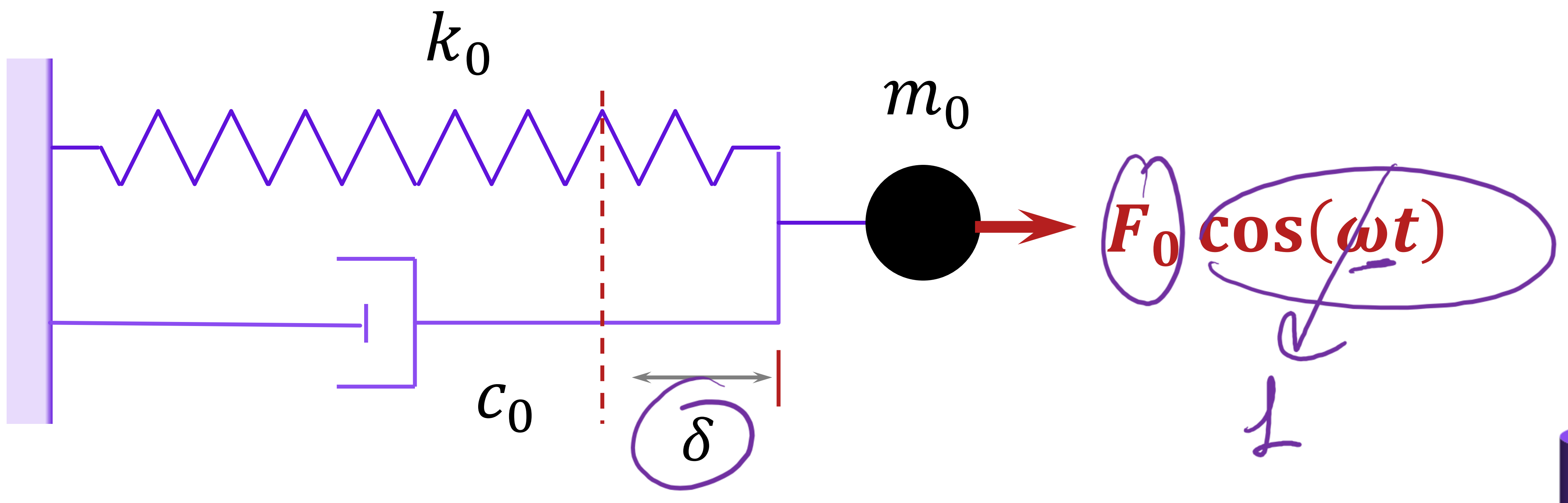
A. 0

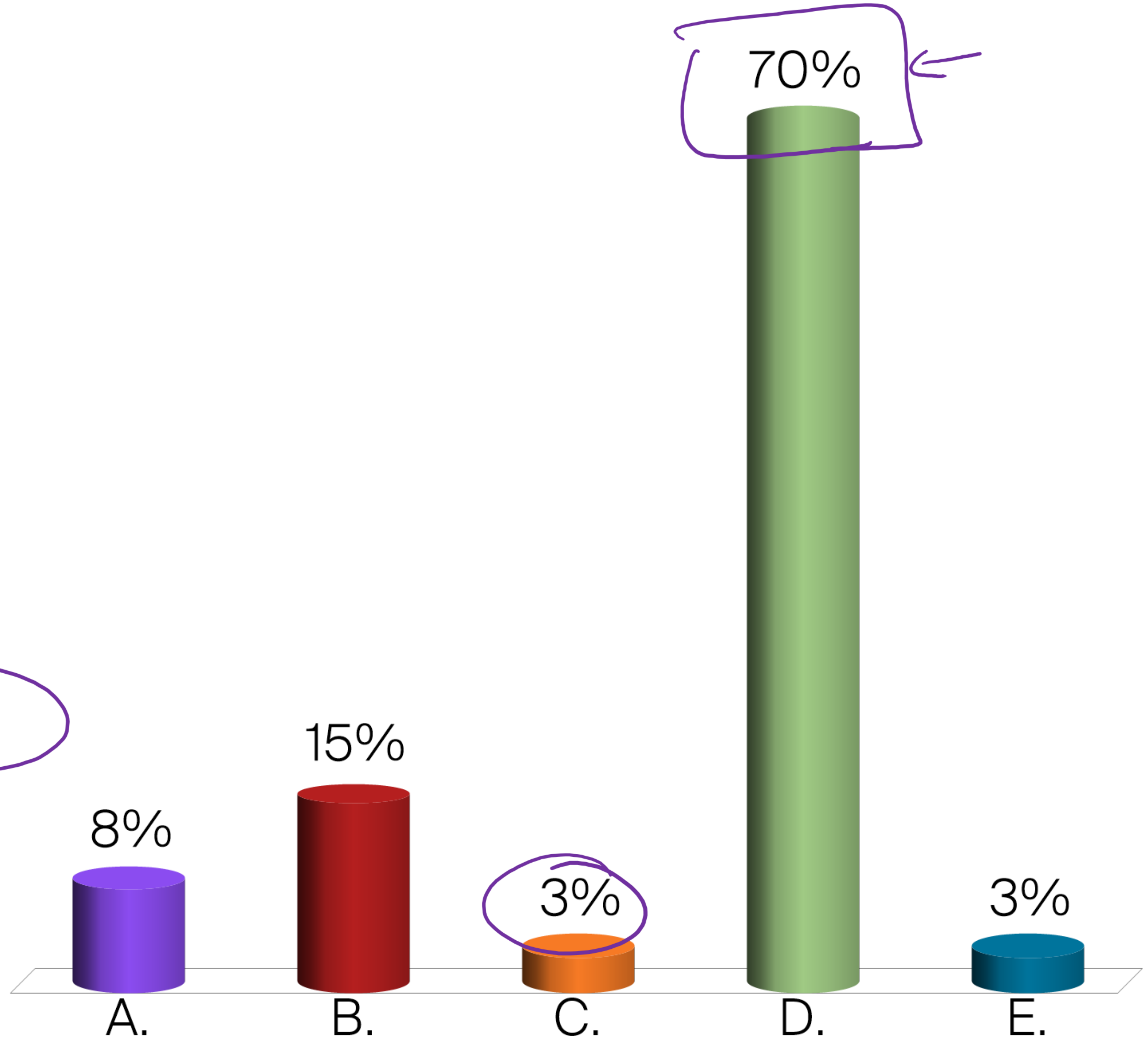
~~B.~~ $\cancel{\delta}$

C. ω_0

D. $\boxed{F_0/k_0}$

E. On ne peut pas savoir





Régime permanent – $x(t)$ si $\omega \to \infty$

A. 0

B. δ

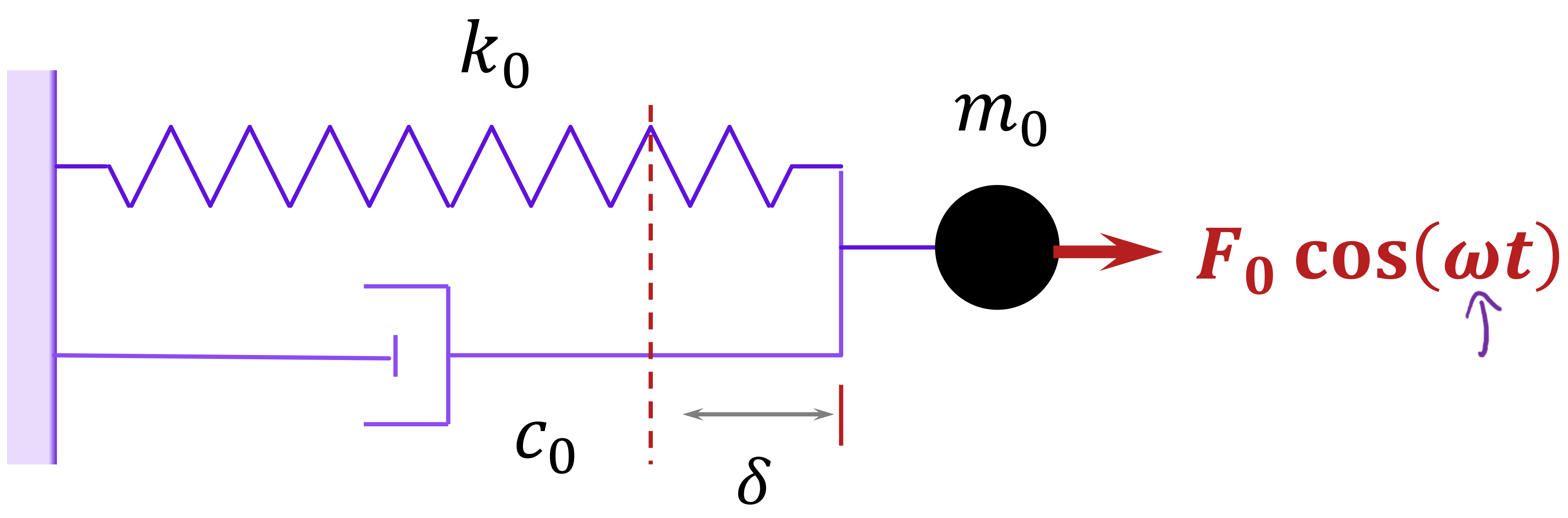
C. ω_0

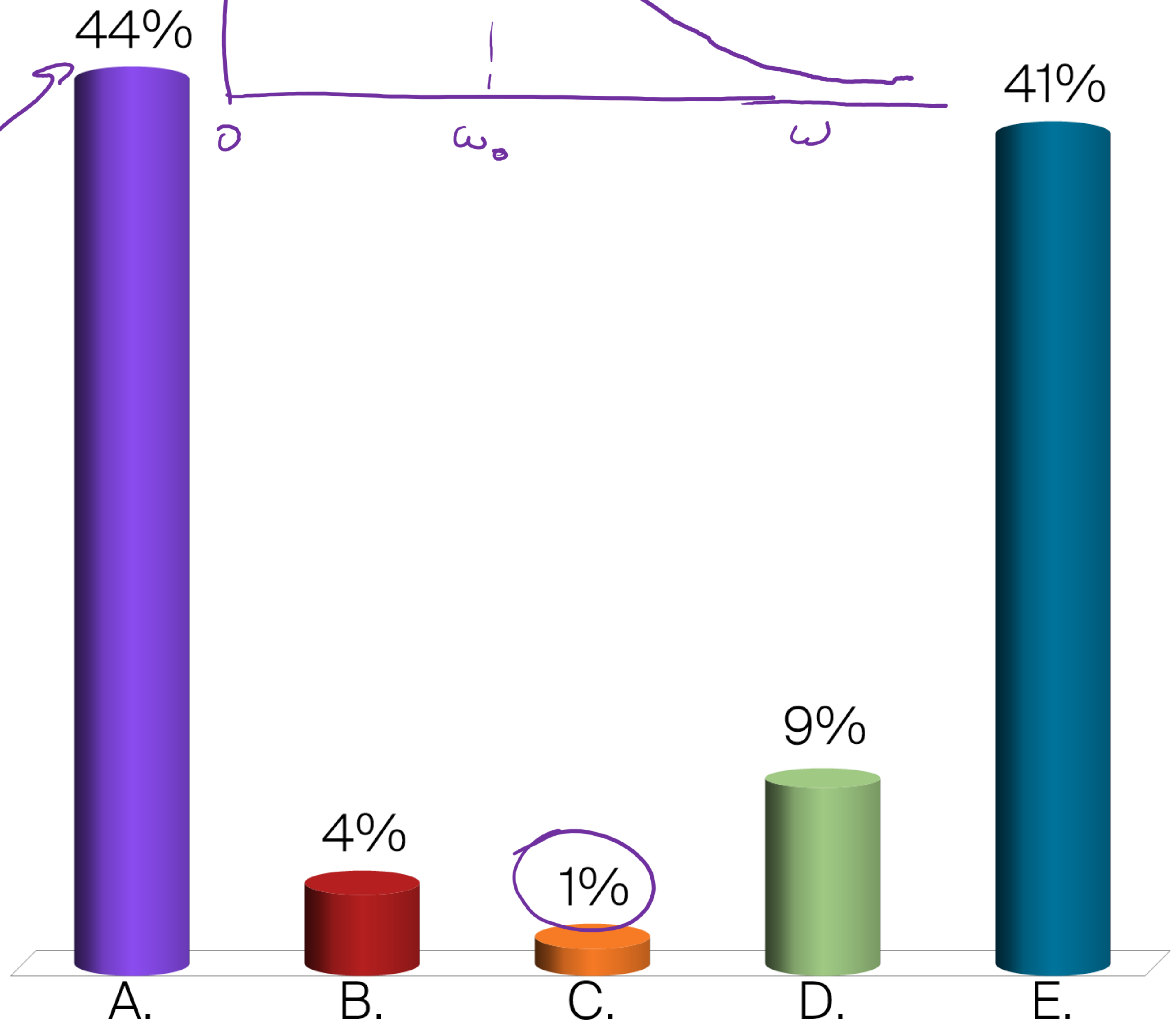
D. F_0/k

E. On ne peut pas savoir

$x(t) = \frac{F_0}{k_0} \cdot \mu \cdot \cos(\omega t - \varphi)$

μ





Régime permanent – Maximum Amplitude

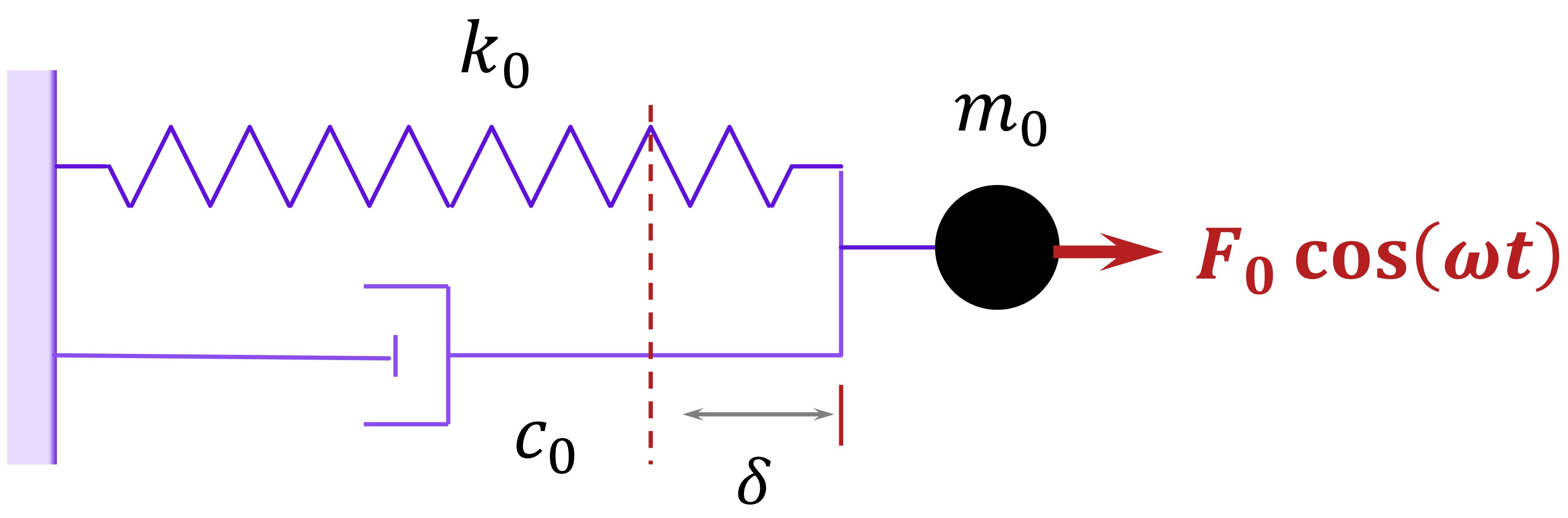
A. δ

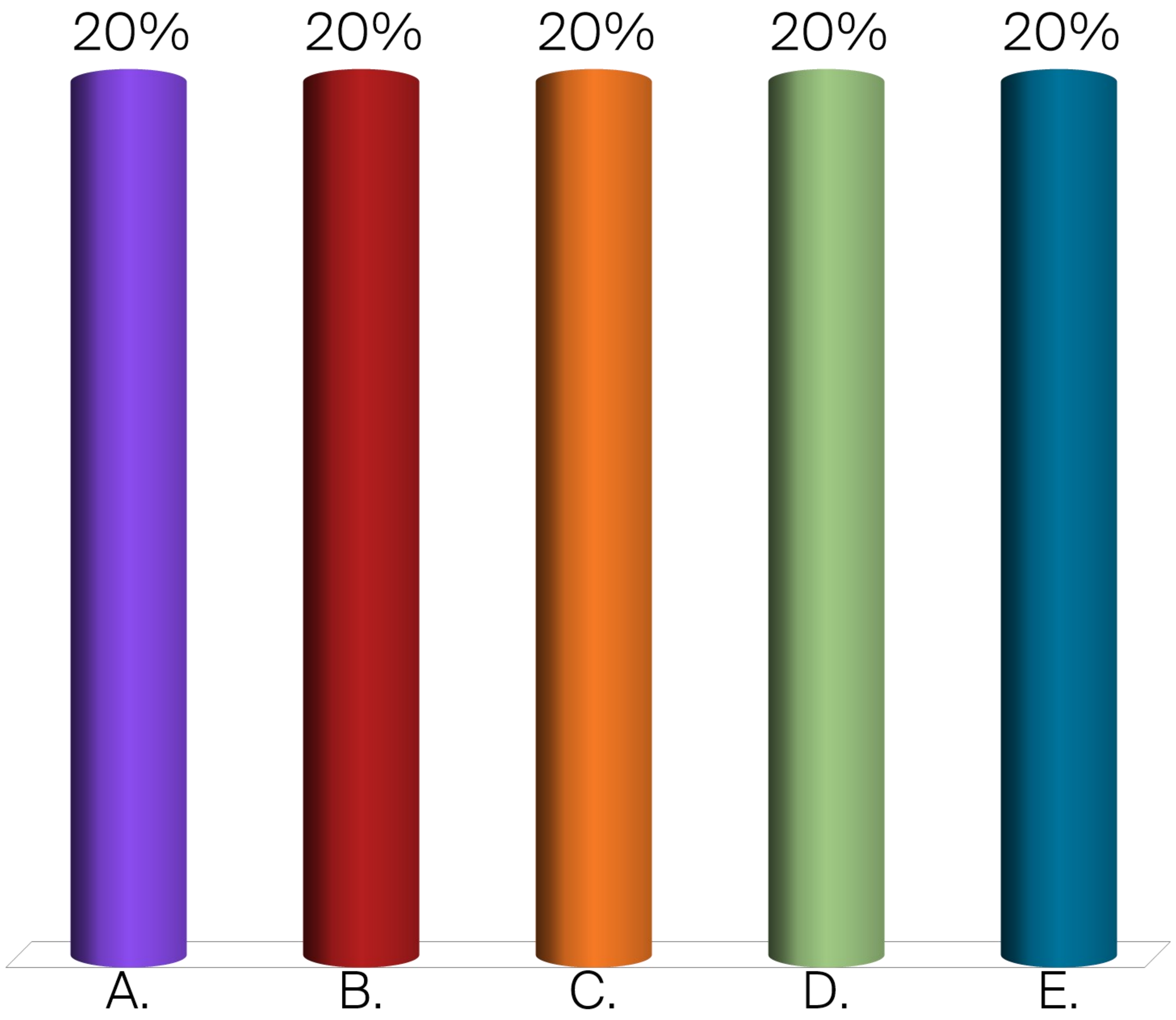
B. $\omega_2 = \omega_0\sqrt{1 - 2\eta^2}$

C. F_0/k

D. $\frac{F_0/k}{2\eta\sqrt{1-\eta^2}}$

E. On ne peut pas savoir





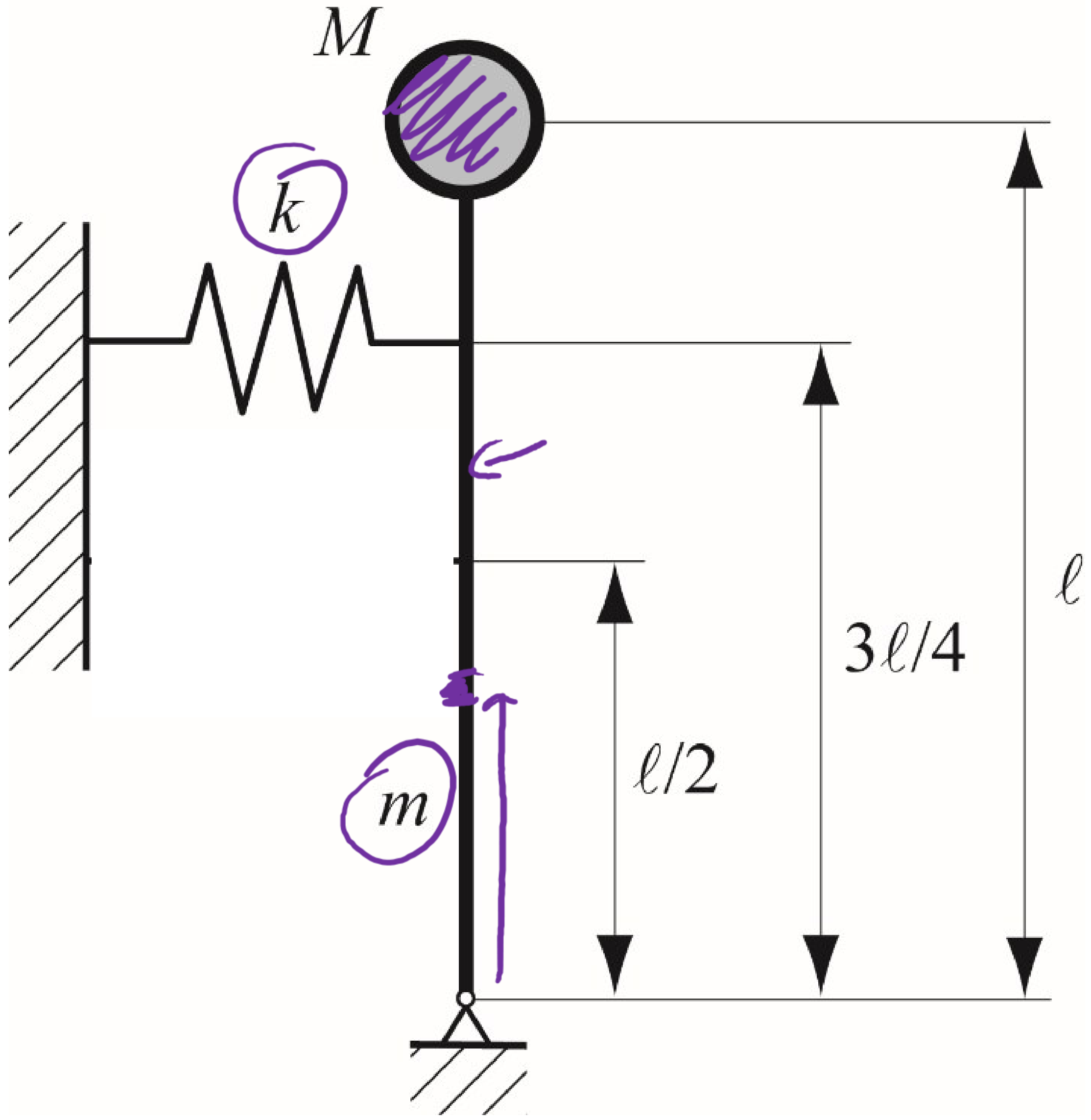
Fréquence ω_0?

A. $\omega_0 = \sqrt{k/(M+m)}$

B. $\omega_0 = 3/4\sqrt{k/(M+3m)}$

C. $\omega_0 = \sqrt{k/(M+m/3)}$

D. $\omega_0 = 3/4\sqrt{k/(M+m/3)}$



$$V = \frac{1}{2} k \left(\frac{3}{4} L \theta\right)^2$$

$$J_{tige} = \int_0^L \frac{m}{L} x^2 dx = \frac{m}{\cancel{L}} \frac{L^{\cancel{3}2}}{3}$$

$$T = \frac{1}{2} M L^2 \dot{\theta}^2 + \frac{1}{2} J \cdot \dot{\theta}^2$$

$$= \frac{1}{2}\left(M + \frac{m}{3}\right) L^2 \dot{\theta}^2$$

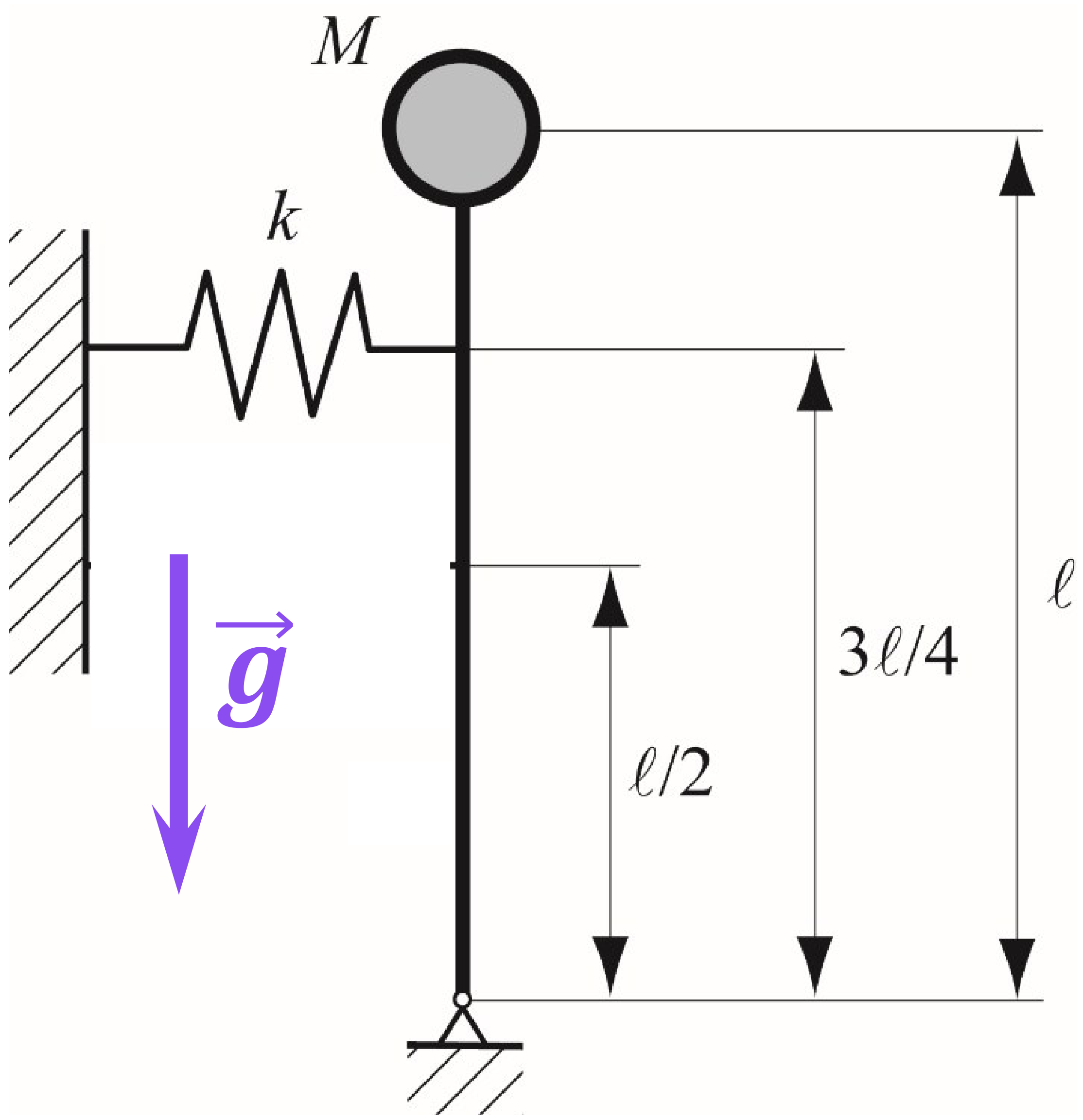
Fréquence ω_0?

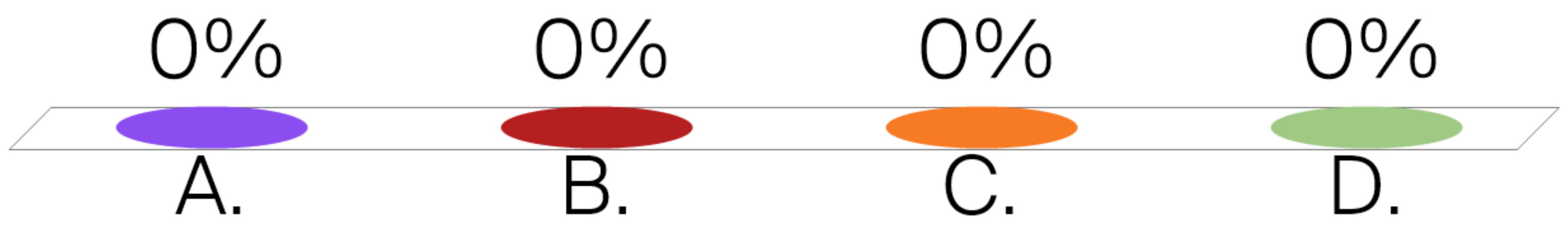
A. $\omega_0 = \sqrt{k/M}$

B. $\omega_0 = 3/4\sqrt{k/M}$

C. $\omega_0 = \sqrt{{}^9/_{16}\,{}^k/_M + g/L}$

D. $\omega_0 = \sqrt{{}^9/_{16}\,{}^k/_M - g/L}$





Fréquence ω_0?

A. $\omega_0 = \sqrt{k/M}$

B. $\omega_0 = 3/4\sqrt{k/M}$

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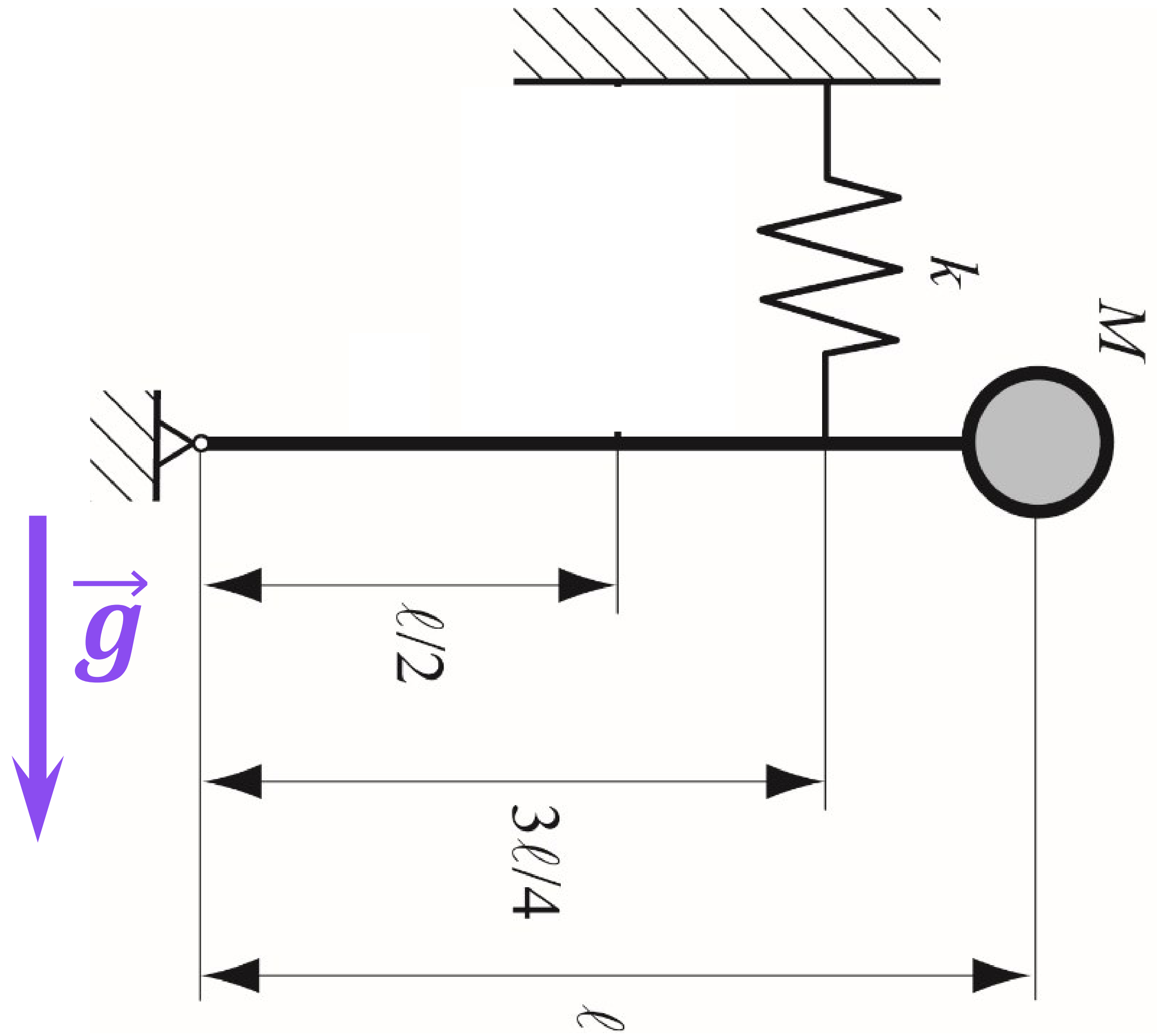

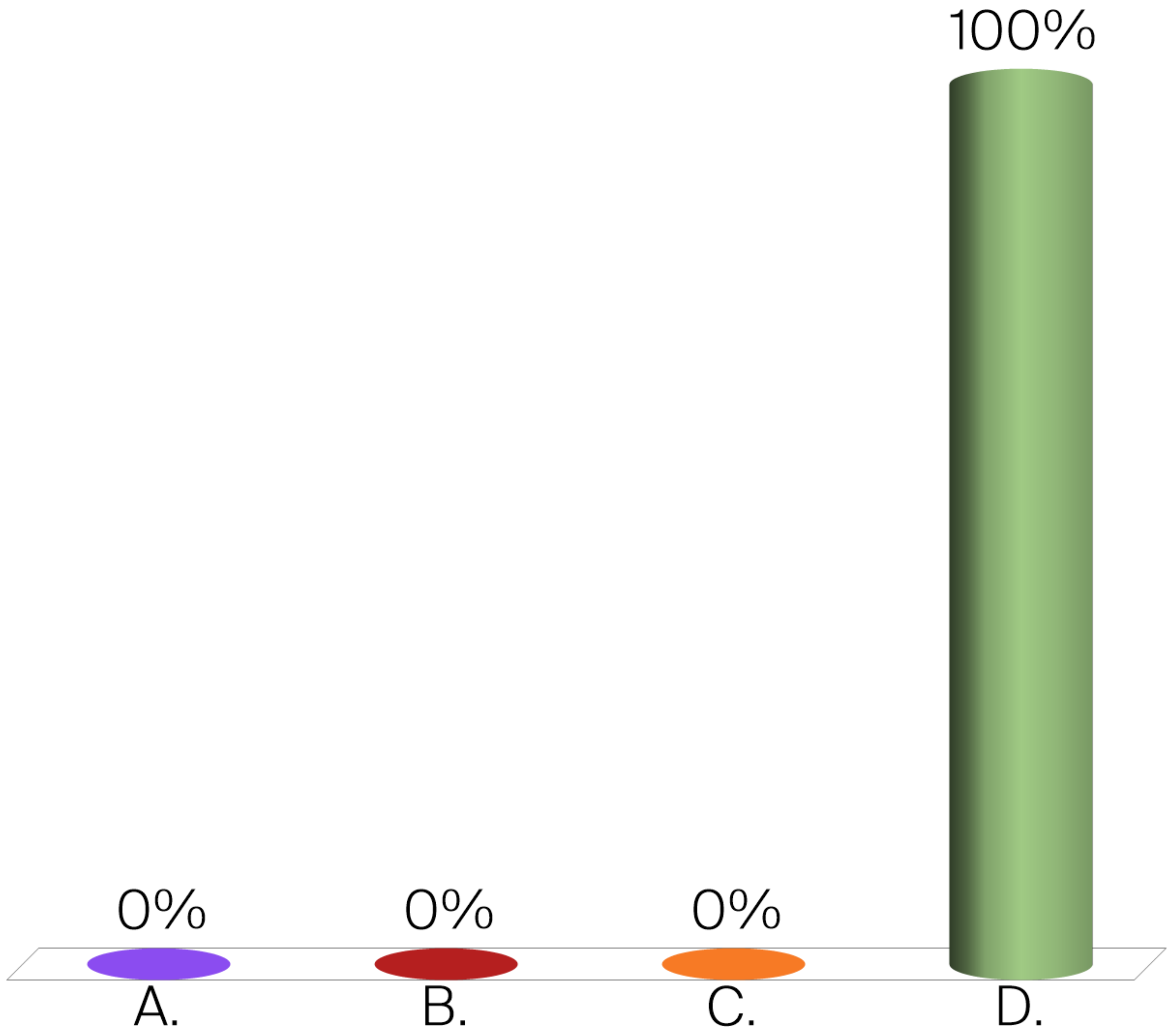

Fréquence ω_0?

A. $\omega_0 = \sqrt{k/M}$

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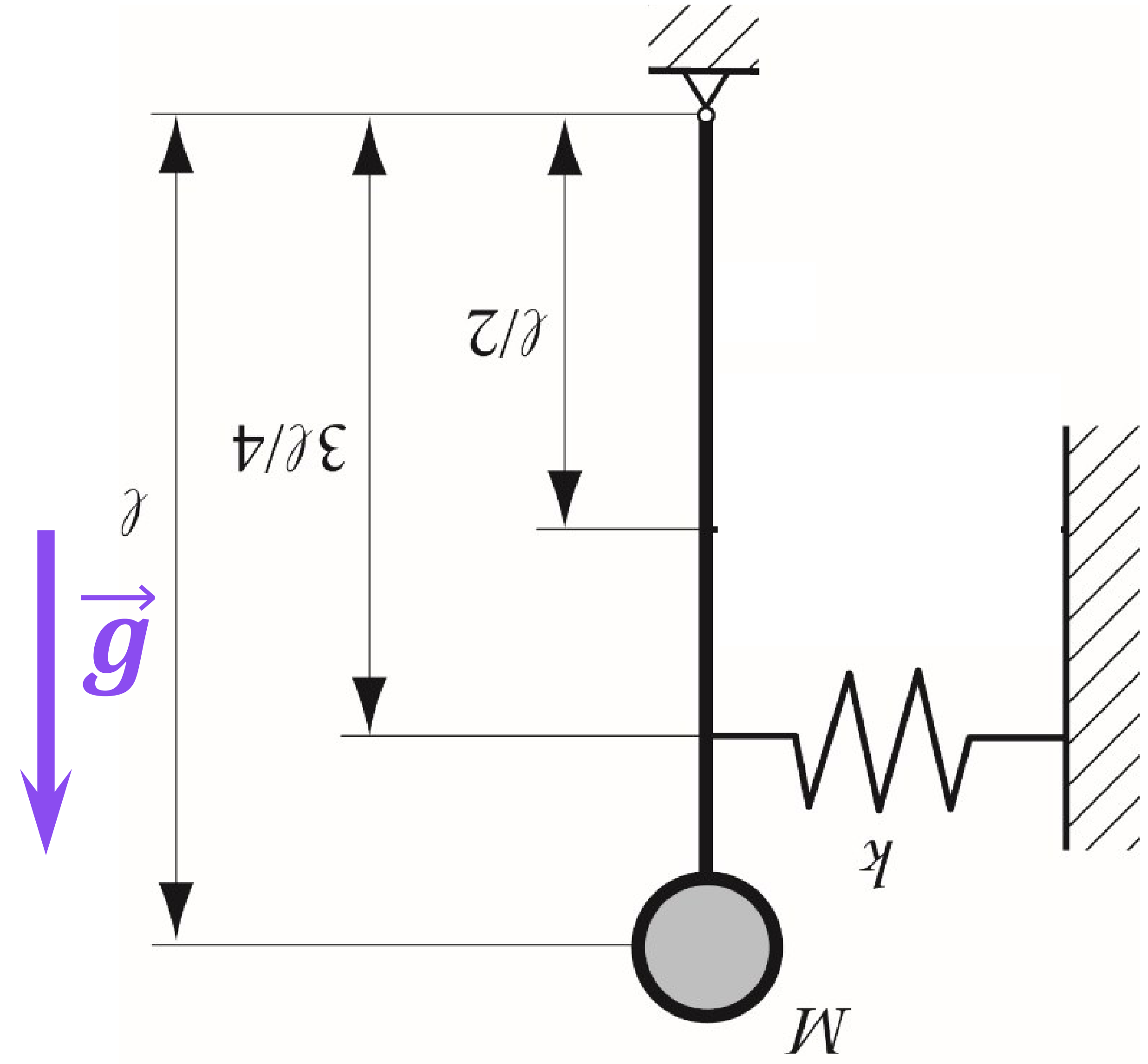

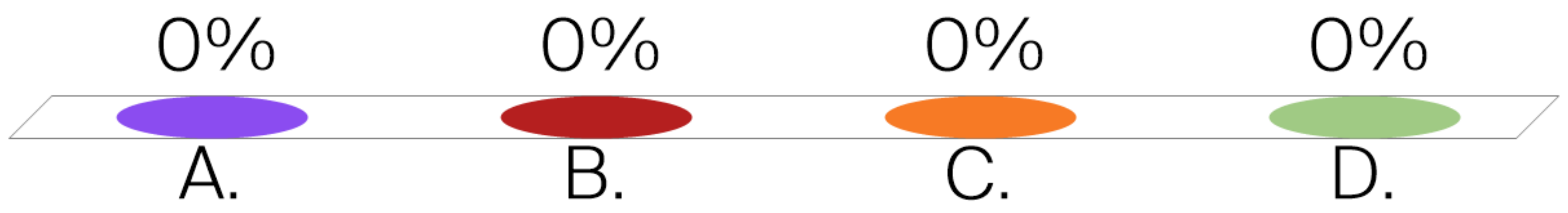

Fréquence ω_1?

$$\omega_0 = \frac{3}{4}\sqrt{\frac{k}{M}}$$

A. $\omega_1 = \omega_0$

B. $\omega_1 = \omega_0\sqrt{1 - \frac{c}{2k}}$

C. $\omega_1 = \omega_0\sqrt{1 - \frac{4c^2}{9Mk}}$

D. $\omega_1 = \omega_0\sqrt{1 - \frac{c^2}{144Mk}}$